Theory and Practice of Auctions

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Abstract

Different auction mechanisms yield different revenue for the seller. The basic auction mechanisms are analyzed. Depending on the assumptions about the characteristics of the bidders, any one of these auction mechanisms may be optimal in the sense of maximizing the revenue for the seller. This is discussed in auction theory of which an overview is given in this paper. Confronting auction theory with practice, it becomes evident, that the characteristics of the good influence the comparative advantage of one auction form over the other. This is hardly discussed in auction theory. In this paper first steps in this direction of investigation are made. Furthermore sources of designing inefficient auctions on the internet are identified and examples are given.

Keywords:

Auction theory, fish auctions, auction practice, internet auctions

Introduction

Auctions are one category of allocation mechanisms among others (Shubik 1970), but a rather general one, if pursued with sufficient sophistication. Several different auction mechanisms are used in practice, ranging from simple internet auctions for used household commodities and other consumer goods, where bidding takes place until a specified time is elapsed, to more sophisticated auction mechanisms for interrelated objects with rules for bidding on more than one object simultaneously and several rounds of bidding with different bidding rules employed in each round. The simultaneous ascending auction for interrelated objects was developed by economists in the United States of America and employed by the government for selling licenses to use the electromagnetic spectrum for personal communication services, like mobile phones and wireless computer networks. The design of the spectrum auctions is described in detail in McMillan (1994). The auction was pursued in summer 1994. The results have been analyzed by several economists, including those already involved in designing the auction. The results and the analysis of the auction are presented in McAffee and McMillan (1996) and Milgrom (2000). The UMTS-auctions, as already conducted in several countries in Europe and about to be held in the U.S.A. have a similiar design.

In economic theory, we distinguish four basic auction rules: first-price sealed-bid, second-price sealed-bid, English and Dutch auction.

In the *first-price sealed-bid auction*, sealed bids are submitted and the seller opens the bids and the bidder with the highest bid wins the object at a price equal to her bid. In economic theory, the reverse form of the first-price sealed-bid auction is regarded as strategically equivalent to the usual first-price sealed-bid auction. The reverse type of first-price sealed-bid auctions are popular in procurement in the business-to-business markets or in tenders used by governments for the procurement of goods and services.

If more than one unit of a homogeneous good is sold simultaneously, each successful bidder will pay a different price for the units she receives. This has some resemblance to perfect price discrimination by a monopolist. Accordingly will use the term *discriminatory auction*, when the first-price sealed-bid auction is employed for several units of a homogeneous good. Each successful bidder receives the units of the good she has stated on her bid at the price she submitted as a bid. This kind of a discriminatory auction is used in practice to sell government securities in countries throughout the world. The U.S. Treasury auctions off securities more than 2 Trillion Dollar worth annually. While the discriminatory auction rule has been used since 1929, however in 1992 the Treasury began to auction off the two- and five-year notes using a uniform-price method (Bartolini and Cottarelli 1997). An example from the agricultural sector of the use of a reverse discriminatory auction are the weekly tenders for grain export licenses. Grain exporting companies submit bids stating the quantity of e.g. wheat together with a restitution claim. If a company has submitted a bid which is accepted by the European Commission, the company will receive the claimed amount of restitution, when exporting the wheat.

In the *second-price sealed-bid auction*, bid are submitted secretly to the seller and the auctioning rule states that the bidder with the highest bid (the lowest cost) will receive (deliver) the good (service) at a price which is equivalent to the second highest bid (the second lowest cost). This kind of auction has been developed by Vickrey (1961) in his seminal paper on auction theory. The bidding rule is hardly used in practice, probably due to the problems of moral hazard associated with this bidding rule and the bidder resistance to truth-revealing strategies (Rothkopf, Teisberg and Kahn 1990). The seller has strong incentives to change the rule after the bids have been submitted secretly. The information advantage could be exploited with opportunistic behaviour of the seller and this is known to the bidders. The incentives for the bidders to take part in such a mechanism increase with their trust in the seller not to behave in an opportunistic way. Accordingly the gains of trade are higher in the case of the trusted seller, like the government. If third parties are able to capture a fraction of the economic rent revealed in the second-price procedure, bidders will be reluctant to follow the optimal strategy, which means to reveal the true valuation of the object.

If not one object, but units of a homogeneous good are auctioned off this way, we will use the term *uniform-price auction* to label the rule, where each winning bidder receives the units of the good at an equal price, regardless of the amount submitted as a bid. Of course this bid has to be higher than or equal to the price to become a winning bid. The empirical research of those treasury security auctions performed as uniform-price auctions since 1992 seem to support the view, that the uniform-price auctions results in higher revenue for the seller than discriminatory auctions, at least in the case of treasury securities (Bartolini and Cottarelli 1997). At first sight this seems to contradict the economic intuition, that price discrimination will result in a higher revenue for the seller than uniform pricing. But this intuition does not take into account the character of the auction as a game. Will will return to this later.

In the *Dutch auction*, a clock or another device is used to decrease the price until the first bid is received. These auctions are popular for flowers, fish and other fast perishable items. Several items are sold sequentially. The bidder with the highest bid (who was the first to push

the bottom to stop the clock) receives the object at the price stated on the clock. Traditionally bidders gathered at the place, where bidding took place. This is about to change. Several European Fish Auctioning Houses linked together in the Pan European Fish Auctions (PEFA.COM) and set up a virtual clock on the internet to link the real auction with a virtual one. As discussed later, the Dutch auctions have some advantages in practice. These advantages are lost, when these auctions are pursued on the internet and bidders no longer gather at one place. Sometimes the Dutch auction is denoted as *descending auction*, because the price decreases during the auction. We will use the term *descending auction* for any auction, be it on the internet or at a real auction place, where the price increases during the bidding process The particular characteristic of the Dutch and the English auction is, that bidders gather at one place to bid in public.

In the *English auction*, like in the Dutch auction, bidders traditionally gather at a place to bid. Bids are raised incrementally. The bidder with the highest bid receives the object. If the increments are infinitely small, the winning bidder has just to outbid the bidder with the second highest bid. Accordingly the price to be paid by the winning bidder is equivalent to the second highest bid submitted during the course of the auction. In contrast to the second-price sealed-bid auction, during the bidding process in the English auction each bidder becomes informed about the bids submitted by the other bidders. The English auction is used to sell antiques or arts and other goods which are scarce or unique. We will use the term *ascending auction* for any auction, be it on the internet or at a real auction place, where the price increases during the bidding process.

To summarize: we will distinguish between auctions where real people gather at one place (English and Dutch auction) and auctions where bids are submitted sealed (first-price sealed-bid and second-price sealed-bid auction). Furthermore we will distinguish between descending auctions (Dutch and first-price sealed-bid) and ascending auctions (English and second-price sealed-bid). In the context of several units of a homogeneous product put up for sale, we will use the terms discriminatory auction for auctions where each successful bidder pays a different price and uniform-price auctions where all successful bidders pay the same price.

The rules of the auction may be rather sophisticated by adding other rules to the bidding process than those already discussed, like in the simultaneous ascending auction used to sell the radio spectrum licenses. This paper will start with the analysis of the basic sets of rules as specified by the so called first-price sealed-bid, second-price sealed-bid, English, Dutch, uniform-price and discriminatory auction rules. Other rules supplementing these rules will be discussed later.

Auction theory is rather advanced; in particular one aspect has been analyzed in detail in the literature: the design of the optimal auction. With some vey few exceptions, for example in the case of auctions for spectrum licenses, economic theory and practice never had an intensive meeting with each other. One aspect has found so far only minor interest in auction theory: why is a certain auction rule used in practice for a particular good and another rule for another good. To make first steps into this direction, we will distinguish here following *categories of goods* to be auctioned:

- homogeneous units of a good, like treasury securities

- heterogeneous units of a good, like flowers or fish

- a unique good, like arts.

Furthermore, we have at least to distinguish between

- fungible or not fungible goods

- perishable or not perishable goods.

We will make a clear distinction between real and virtual auctions. In real auctions bidders gather at one place and have the opportunity to inspect the goods put up for sale, whether the English or the Dutch auction system is used. The bids becoming available in the process of bidding are public information for the bidders. In postal auctions, first- or second-price, bidders submit sealed bids. There is no information available on the bids of the other bidders. In ascending bid auctions as pursued on the internet, sometimes information on the bids submitted by the other bidders is available, sometimes not. A particular sensitive question is how much information on the good and the seller becomes available for the bidder in internet auctions. We will discuss this important point for practical implementation of auctions on the internet more in detail. But before doing so, we will give a brief review of the results of auction theory.

In the literature, several approaches have been chosen to analyze auctions. Here we will regard as a benchmark model the auction as a mechanism (Molho 1997), or, what is equivalent, as a static game with incomplete information (Gibbons 1992). Furthermore, without loss of generality, we will employ some restrictive assumptions to keep the exposition easy to understand even for those being no experts in auction theory.

The benchmark auction model

The auction model that is the easiest to analyze is based on the following four assumptions (McAffee and McMillan 1987, p. 706):

- the bidders are risk neutral

- the independent privat value assumption applies, that is bidders valuations are independent

- the bidders are symmetric, that is that the distribution underlying bidder's valuations is identical for all bidders

- payment is a function of bids alone, that means entrance fees etc. are excluded from consideration

Furthermore we will simplify the exposition by considering

- only two bidders labeled i = 1,2 and

- only one item is auctioned off, that means it is a "one-shot game".

Each bidder attributes a value to the item. The value of the item to bidder i is denoted as v_i . If bidder i gets the item at a price p, then her payoff is v_i - p. The gain of trade for the seller is the difference between his valuation attributed to the object and the price paid for the good: p - v_s. At this stage of analysis let us assume that the valuation of the seller is $v_s = 0$. We will discuss positive reservation prices for the seller later. The seller determines the auction rules and implements the auction. Each bidder knows, that the price she has to pay depends on her

bid and the bid of the other bidder. Each bidder has incomplete information about the bid of the other bidder when submitting her own bid. While the subjective value v_1 is private information of bidder 1, the subjective value of the other bidder v_2 is private information of bidder 2. The game is transformed with the Harsanji-transformation to a game of complete but imperfect information by introducing a move of nature. We will assume, that nature draws the valuations independently for each bidder from an identical uniform probability distribution between 0 and 1. Each bidder receives the information on her valuation, but not on the valuation of the other bidders and the seller. The rules of the game together with the probability distribution from which nature draws the valuations for the bidders are common knowledge among the bidders and the seller. Each bidder knows her type (valuation), after nature has drawn, but not the type (valuation) of the other bidder.

The first step of this game consists of an action taken by "nature" in assigning a type to each bidder. In the second step each bidder will choose an action. The action space consists of bids. These bids act as signal to the seller. Of course, the bid of a bidder i may be different to her valuation v_i. We model the auction here as a static game and accordingly the action space is identical with the strategy space and consists for each bidder of bidding a value between 0 and 1. The distribution is common knowledge among bidders and the seller and accordingly bids outside this range will not occur in equilibrium. The bid is the signal and depends on the valuation of the respective bidder. The seller receives the bids submitted from different bidders and determines the winning bidder and the price to be paid for the item (according to the rules stated by the seller before bidding takes place).

Each bidder is assumed to choose a strategy maximizing her payoff, given her type. The probability that bidder i = 1 will get the object depends on the bid submitted by player 1 and the other player i = 2. If players collude, they would bid 0 or an infinitesimal amount and allocate the object between them both according to binding rules specified before submitting the bid. At this stage of analysis, we will assume no cooperation between bidders. Collusion is discussed later. In a competitive environment each bidder is aware that the price she will have to pay, if submitting the highest bid, depends on the bid submitted by the other bidder. The seller will maximize his revenue and therefore allocate the item to the bidder with the highest bid submitted. The auction mechanism specifies how bidding takes place and how the price to be paid to the seller is determined given the bids received.

In order to summarize the assumptions of the benchmark model:

two bidders i = 1,2 and one object. The bidder i has payoffs:

$u_i = v_i - p$ when i gets the object
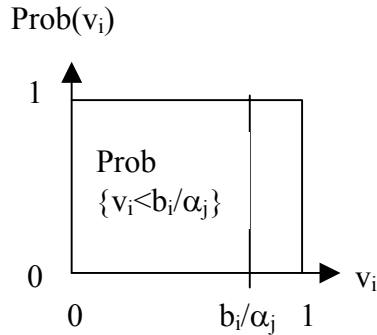
$u_i = 0$ otherwise

and the seller s has payoffs:

$u_s = p - v_s$ if the object is sold

$u_s = 0$ otherwise,

where v_i has idential and uniform distribution on [0,1], as presented in **figure 1**.

Figure 1
The distribution of the valuations of bidders

Prob(v_i)

Equilibrium bidding strategy in the first-price sealed-bid auction

In a first-price sealed-bid auction, the bidder with the highest bid receives the item at a price equal to his bid. Each bidder i is aware, that increasing her bid will increase the probability of winning, but on the other side decrease the price and accordingly the payoff received when winning. We will consider two bidders, labeled i and j. The pair of strategies (b(v_i), b(v_j)) is a Bayes-Nash equilibrium of this static game of incomplete information, if for each v in [0,1], b (v) solves:

$$\text{Max} \ E(v_i - b_i) * \text{Prob}\{b_i > b_j(v_j)\} + 1/2 * E(v_i - b_i) * \text{Prob}\{b_i = b_j(v_j)\} \tag{1}$$

$$b_i$$

The first term is the expected payoff when winning with the submitted bid times the probability that the submitted bid is higher than the bid of the other bidder j. She second term is the expected payoff in the case of a tie times the probability that a tie will occur. We will assume that the seller throws a dice to determine the winning bidder when both submitted bids are equal amount.

To keep things simple, bids may take any real value (with sufficient precision) between 0 and 1 and accordingly the probability of a tie approaches 0. With this assumption the equation (1) reduces to

$$\text{Max} \ E(v_i - b_i) * \text{Prob}\{b_i > b_j(v_j)\} \tag{2}$$

$$b_i$$

We will keep the exposition simple and follow Molho (1997) in looking only for a linear equilibrium of the form:

$$b_i = \alpha_i * v_i \tag{3.1}$$

where $1 \geq \alpha_i \geq 0$

and

$$b_j = \alpha_j * v_j \tag{3.2}$$

where $1 \geq \alpha_j \geq 0$.

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We will use equation (3.2) to simplify equation (2):

Max E(v_i - b_i) $*$ Prob{$b_i > \alpha_j * v_j$ }

   b_i

or

Max E(v_i - b_i) $*$ Prob{$b_i/\alpha_j > v_j$ }   (4)

   b_i

We will employ the assumption of a uniform and identical distribution of the bidders' valuations in the range from 0 to 1 to simplify further. With this distribution the probability that a true valuation is smaller than, let us say 2/3, is just 2/3. The second bidders valuation v_j is a random draw from this distribution. Accordingly the probility that bidder j has a valuation $v_j < b_i/\alpha_j$ is b_i/α_j (compare figure 1). Therefore equation (4) becomes:

Max E(v_i - b_i) $*$ b_i/α_j (5)

   b_i

To find that bid, that maximizes this expected return, we calculate the first-order condition:

(v_i - b_i)$*1/\alpha_j$ -b_i/α_j = 0

or

(v_i - $2*b_i$)$/\alpha_j$ = 0

Which solves for the optimal bid of player i:

$b_i = 1/2 * v_i$ (6)

The maximization problem is symmetric for player j:

$b_j = 1/2 * v_j$.

Each bidder maximizes her expected return by placing a bid, which is exactly half of her valuation of the good. For example, if the bidder has a true valuation of 2/3, she will bid 1/3. The second bidder faces a symmetrical problem and accordingly her optimal bid is also half of her true value. In this Bayes-Nash equilibrium among two bidders no bidder can do better than bidding half of her valuation, given the other bidder follows the same strategy.

The analysis has been generalized by Vickrey (1961) for the case of N bidders, where N may take any positive integer number. The case of two bidders is a special case of this more general model. The general model results in the following optimal bidding strategy for each bidder i:

$b_i = (N-1)/N * v_i$ (7)

If N = 1, that is with just one bidder, the bidder bids just the smallest amount, the seller will accept, that is 0, assuming a valuation of 0 attributed by the seller to the item. In the case of

two bidders, the result above is replicated, that each bidder will bid just half of her valuation. If N becomes larger, each bidder will tend to bid closer to her true valuation.

We have presented this little auction model here, because it is easy to understand and has already the theoretical richness to discuss (but not prove) the most interesting results of auction theory for the practice. This will be done later in this paper.

Equilibrium bidding strategy in the second-price sealed-bid auction

The bidding rule has to be changed in the model presented above to accommodate for the second-price sealed-bid auction rule. The bidder with the highest bid will receive the item at a price equal to the second highest bid submitted. At first sight this seems to reduce the price to be paid by the successful bidder compared to the first price sealed bid auction. But this is not the case, because the bidding strategy depends on the rule for determining the price to be paid for the object. In the second-price sealed-bid auction, bidders have a dominant strategy, which is not the case in the first-price sealed-bid auction and accordingly the equilibrium is rather easy to analyze. The dominant strategy is due to the fact, that the price the bidder with the highest bid has to pay, is independent of her submitted bid. The probability to win the object increases with the bid submitted, but not the price to be paid for the object. The expected payoff is maximized by bidding the true valuation, because equation (2) becomes in the case of the second-price sealed-bid auction:

$$\text{Max } E(v_i - b_j) * \text{Prob}\{b_i > b_j(v_j)\} \qquad (8)$$
$$b_i$$

The expected price, bidder i has to pay, is independent of her bid in the second-price sealed-bid auction and accordingly the expected payoff can be maximized by maximizing the probability of winning, as long as the payoff is positive. This probability will increase with the amount submitted as a bid. The bid submitted has no direct influence on the price to be paid and accordingly the bid submitted is raised up to the bidder's valuation of the item. In the second-price sealed-bid auction it is a dominant strategy to bid the true value. Bidding the true value is the always best answer, regardless of the strategy used by other bidders.

Equilibrium bidding strategy in the Dutch auction

If bidders are rational, the general assumption in auction theory, then it holds that the Dutch and the first-price sealed-bid auction rule are strategically equivalent. Accordingly the Bayes-Nash equilibrium strategies are the same in these two auctions. This abstracts from the fact, that traditionally in Dutch auctions bidders gather at a real place. As modeled in auction theory, the bidding process in the Dutch auction conveys no new information for a bidder. In order to calculate the optimal bid, the bidder already takes the information on the distribution into account by submitting a strategic bid. If the clock reaches the number representing the optimal bid, the bidder having this bid will press the bottom to stop the clock and to submit her bid. Inspecting the good, talking to other bidders etc. is important in this environment. Gathering at one place incurs travel cost (including time spend for traveling). The benefit of gathering at a real place seems to outweigh these costs incurred in pursuing Dutch auctions, otherwise first-price sealed-bid auctions would have been used as the alternative with lower cost incured. The Dutch auction is popular for perishable goods, flowers and fish.

The Dutch auction has one important advantage compared to other auctions, not taken care of in auction theory: it is a very fast mechanism to allocate a good. The clock is used as the device to facilitate very fast bidding processes. Only one bid is submitted and as such it is information efficient with regard to the information needed to allocate the good to the highest bidder. Any other mechanism, like the ascending bid auction, will require more information to be exchanged before the allocation takes place. This explains why this auction mechanism is used for perishable goods which have to be allocated as soon as possible.

Equilibrium bidding strategy in the English auction

The English auction seems to be strategically equivalent to the second-price sealed-bid auction. But this holds only for the special case of independent and identical distributed valuations of the bidders. If the bidders are not sure about the true value of the object, the information conveyed during bidding may have an influence on bidders' valuations of the object to be sold. If we assume that bidders valuations are affiliated, other results will emerge, as discussed later.

Assuming independent private values, the English auction is strategically equivalent to the second-price sealed-bid auction. Compared to the second-price sealed-bid auction, the cost participating in the English auction is higher due to the need to gather at a place. This has been discussed already in the case of the Dutch auction.

The English auction is used to sell antiques and arts and other rather unique objects. These goods are not very fungible, that means that they hardly can be described sufficiently to the seller without visual inspection and accordingly visual inspection may be important to evaluate the value of the good.

Equilibrium bidding strategy in the uniform-price auction

The uniform-price auction pursued by one seller for several units of a homogeneous product is theoretically equivalent with the microeconomic standard model of a monopolist facing a given demand curve. The optimal solution for the seller is choosing the price where marginal revenue equals marginal cost. One price is charged for all units of the good.

The uniform-price auction gives information to the seller about the demand curve, because bids reflect the true valuations of the buyers. With the uniform-price auction rule the seller has committed not to use this information in determining the prices, but this information may be very important for the seller for other purposes. For example in the case of export tenders for wheat, if the European Commission would change the auction system from a discriminatory auction to a uniform-price auction, the demand curve and the price elasticity of demand for wheat would become available. This is an important information needed in designing agricultural policy, accordingly it seems to be more efficient to conduct these tenders as uniform-price auctions (Becker 1989).

Equilibrium bidding strategy in the disriminatory auction

The discriminatory auction for several units of a homogeneous good seems on first sight to be equivalent to monopolistic price discrimination. With a given demand curve, it is obvious that price discrimination will result in a higher revenue for the seller. But in the case of a discriminatory auction, the bids submitted do not reflect the true demand curve. Such a bid

curve becomes available with each of the four auction types except the Dutch auction, where only the winning bid is available after the end of bidding. Bidders may submit strategic bids depending on the assumed bidding behaviour of the other bidders in some of these auctions. In other auctions bidders may have an incentive to reveal with the submitted bid their true value of the object. Only in the later case, the bids reflect the true demand curve of the bidders in the sense the demand curve is defined in standard microeconomic theory.

The revelation principle

Employing the revelation principle, the above results are recasted in the mechanism design approach. The revelation principle states that any Bayes-Nash equilibrium of any Bayesian game can be represented by an incentive compatible direct mechanism.

The optimal strategies for the two players in the static game of incomplete information derived above for the first-price sealed-bid and Dutch auction form a Bayes-Nash equilibrium. The optimal strategies for the second-price sealed-bid and English auction form a dominant strategy equilibrium. A dominant strategy equilibrium is much more stable than a Bayes-Nash equilibrium because if a player has a dominant strategy, beliefs about the other players valuations do not play a role in strategic behaviour. Representing the model as an incentive compatible direct mechanism will lead to the same results but other important insights become available. In an auction mechanism, the goal of the seller is to maximize his revenue, given the participation constraint, that each type of player has an incentive to participate in the mechanism, and the incentive compatibility contraint, that each type of player has an incentive to reveal thruthfully her type, id est to bid her valuation. This is the representation as an incentive-compatible direct mechanism.

A seller who wishes to maximize his expected revenue has many auction mechanisms available. Specifying the many different auction mechanisms the seller should consider could be an enormous task. Fortunately, the seller can use the revelation principle to simplify this problem in two ways. First, the seller can restrict attention to those mechanisms, which are direct mechanisms. A direct mechanism is a static Bayesian game in which each player's only action is to submit a claim about her type. Second, the seller has to investigate only incentive compatible mechanisms, that is those direct mechanism in which truth-telling is a Bayesian Nash equilibrium. Given any feasible auction mechanism, there exists an equivalent incentive compatible direct mechanism, which gives to the seller and all bidders the same expected utilities as in the given mechanism. The revelation principle guarantees that no other auction has a Bayes-Nash equilibrium that yields the seller a higher expected payoff, because such an equilibrium of such an auction would have been represented by a truth-telling qulibrium of a direct mechanism, and all such incentive compatible direct mechanisms were considered as solutions of the programming problem. Myerson (1979) shows for a more general case, that the symmetric Bayes-Nash equilibria analyzed above are equivalent to this payoff-maximizing thruth-telling equilibrium. This implies, that, given the assumptions above, there exists no other auction mechanism which will result in higher revenue for the seller, than the second-price sealed-bid auction.

This is trivial for the case of the second-price sealed-bid auction. If the seller will employ the rules of the second-price sealed-bid auction, thruth-telling is the dominant strategy. But rather sophisticated to understand for the rules of the first-price sealed-bid auction. Consider a direct mechanism, in which the seller simply asks the bidders to report to him their true valuations of the object. Usually it will be in the interest of the bidders to lie. Suppose, however, that the

seller offers the following mechanism: the bidder who reports the highest valuation v_i will win the object and pay a price:

$$p_i = (N-1)/N * v_i \tag{9}$$

Compare this equation (9) with equation (7) stating the optimal bidding rule for a bidder in the first-price sealed-bid auction. The trick is, that the seller announces with this price rule, that he can and will undertake the same calculations which are done by bidders in submitting the bids in the first-price sealed-bid auction to determine the price for the bidder with the highest bid. In this case the best strategy for the bidder is to submit as bid her true valuation. Due to this mechanism it is in the interest of the bidders to reveal their type. If a bidder is lying to the seller about her type (valuation of the item), this means lying to herself.

Revenue for the seller

In the second-price sealed-bid and English auction for one item the bidder with the highest bid will receive the item at a price equal to the second highest bid. Because truth-telling is a dominant strategy in these two auction mechanisms, the bidder with the highest valuation will receive the item at a price equal to the valuation of the bidder with the second highest valuation.

In the first-price sealed-bid and Dutch auction, the bidder with the highest valuation will receive the object at a price equal to his bid. In the first-price sealed-bid and in the Dutch auction bidders will submit a bid depending on the numbers of other bidders. If just two bidders are present, each bidder will submit half of his true reservation price as a bid.

In the case of two bidders and one object, the revenue for the seller with the first-price sealed-bid or Dutch auction is equal to $1/2*v_h$, where v_h denotes the value for the bidder with the higher valuation of the object. With the second-price sealed-bid and English auction, the seller will receive v_l, where v_l denotes the value to the bidder with the lower valuation. At first sight it is not obvious, which of these two values is higher, either $1/2*v_h$ or v_l, or whether they are equal.

The expected value of the highest and the lowest of two draws from the uniform distribution will divide up the horizontal distance 0 to 1 into three equal segments, as depicted in the **figure 2**.

Figure 2
Expected value of the low and of the high valuation in the case of two bidders

Prob(v_i)
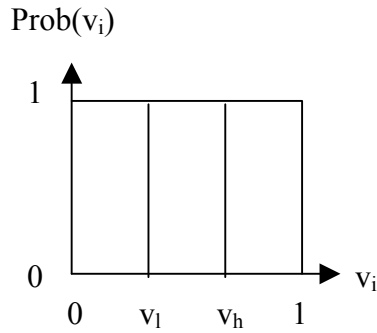
Accordingly, the expected value of v_l is qual to 1/3 and that of v_h is equal to 2/3:

$E(v_l) = 1/3$ and $E(v_h) = 2/3$.

In the second-price sealed-bid auction and in the English auction, the expected price is

$E(p) = E(v_l) = 1/3$.

In the first-price sealed-bid auction or in the Dutch auction, the expected price is:

$E(p) = 1/2 * v_h = 1/2 * 2/3 = 1/3$.

This shows, that in the case of two bidders drawing their valuations independently from a uniform distribution between 0 and 1, the expected revenue for the seller is the same with any of these four auction mechanisms.

This result can be generalized to more than two bidders. With N bidders, the distance between 0 and 1 is divided into N+1 segments. The expected value of the highest valuation is then

$E(v_h) = N/(N+1)$

analogous to $E(v_h) = 2/3$ in the two bidder case. Given the optimal bidding strategy in the first-price sealed-bid auction and the Dutch auction, the expected price is

$E(p) = [(N-1)/N] * E(v_h) = [(N-1)/N] * [N/(N+1)] = (N-1)/(N+1)$ (10)

In the second-price sealed-bid and English auction, the expected price is the expected value of the second highest valuation, which is simply (N-1)/(N+1), that is, it is one segment down from the highest valuation. Thus, the expected price is:

$E(p) = (N-1)/(N+1)$ (11)

which is equal to the expected price with the first-price sealed-bid auction and the Dutch auction.

In order to keep the exposition as simple as possible, we assumed a uniform distribution of valuations between 0 and 1 and only two bidders. The revenue equivalence theorem holds more generally for any distribution of potential valuations, if buyers are symmetric , have independent valuations and if the distribution has a monotone hazard rate. If discrete valuations are assumed the last condition is not satisfied (Fudenberg and Tirole 1996, p. 253).

Reservation price for the seller

It is clearly in the interest of the seller to announce a reservation price to the bidders higher than 0. This has to be taken into account in designing auctions which maximize expected revenue for the seller. In our exposition of the benchmark model we concluded, that with one bidder, the bidder will just bid the valuation of the seller, which was assumed to be equal to 0:

$u_s = 0$.

This assumption was employed to keep the exposition simple. Applying the revelation principle it can be shown, that for the benchmark model presented above the optimal auction has following characteristics: if the expected price to receive given the bids of the bidders is lower than the reservation price, than the seller refuses to sell the object; otherwise he offers it to the bidder with the highest valuation at a price equal to the second highest valuation or the reservation price, if the later is higher than the former. The seller should set the reservation price r_s higher than his own true value u_s of the object. In the case of a uniform distribution of bidders valuations the reservation price of the seller maximizing the expected revenue is just the average of the seller's valuation of the item and the highest possible valuation a bidder could have. This means in the case of independent and uniform distributed bidders' valuations between 0 and 1, that

$r_s = (u_s + 1)/2$

which becomes for $u_s = 0$:

$r_s = 1/2$.

Note, that the optimal reservation price is independent of the number of bidders. If N increases, the reservation price will only become less binding for the expected price, but itself will not change. Furthermore, the optimal reservation price is the same in all auctions regarded.

In the case of one bidder, the reservation price will bring some kind of competition into the game. In the case with two bidders regarded, the reservation price will become binding in the second-price sealed-bid and English auction in some cases and in others not In the first-price sealed bid and Dutch auction, those bidders having a higher valuation than the reservation price of the seller but submitting a lower bid otherwise will increase the bid up to the reservation price of the seller. This advantage has to be compared with the disadvantage of setting a reservation price. It is possible, that the bidder with the highest valuation has a valuation that lies between the valuation of the seller u_s and the reservation price r_s. In this case the monopolistic seller loses the sale even though the bidder would be willing to pay more than the good is worth to the seller. The optimal reservation price is trading off this disadvantage with the advantage of increasing competition.

If sellers use the optimal reservation price, the revenue equivalence theorem still applies. Furthermore, the revenue for the seller is maximized. The optimal reservation price is just the

expected value of the item to be sold to the bidders, given the valuation of the seller. With a uniform distribution of bidders' valuations, the optimal reservation price divides the distance between the valuation of the seller for the good and the highest valuation into two equal segments.

The benchmark model presented above with two bidders is not an optimal auction, unless supplemented by the optimal reservation price of the seller.

If the distribution, from which the bidders' valuations are drawn, is not continuous but discrete, some problems with the benchmark model occur, because the monotone hazard rate assumption does no longer holds. The problem with discrete valuations is, that the high valuation type receives an unnecessarily high rent. The seller can increase his revenue (Fudenberg and Tiroel p. 288) while still inducing buyers to bid their valuations if he specifies the following rule: when one bidder bids b_i and the other bidders bid b_j -let us denote with b_i the lower bid - then the high bidder receives the good at price $b_i + (b_j - b_i)/2$. This has some resemblance with the optimal reservation price and leads in the two bidder discrete case regarded above to the same price as with the optimal reservation price. The optimal reservation price has found with this rule it's counterpart in the mechanism design framework.

Number of bidders and collusion

In the framework developed above it beomes clear that the expected price will increase with the numbers of bidders for any auction. The expected price in each of the four actions regarded so far is, as already derived above, equal to:

$$E(p) = (N-1)/(N+1) \tag{10}$$

Accordingly the expected price will increase with N, the number of bidders.

In the first-price seladed-bid and Dutch auction, the optimal bid, derived above, is a function of the numbers of bidders:

$$b_i = (N-1)/N * v_i \tag{7}$$

and will increase with the numbers of bidders and accordingly the price received by the seller. In the second-price sealed-bid and English auction the expected value of the second highest bid increases with the number of draws from the probability distribution of valuations. Consider the following simple discrete example of a dice. If only one dice is thrown, than the probability of let us say a five, is 1/6. If two dices are thrown, the probability that at least one dice shows a five increases to $2 * 1/6 = 1/2$. The more dices thrown, the higher the probability of a high value. This leads to the result, that as the number of bidders approaches infinity, the price tends to the highest possible valuation. In the model presented above, this is just the upper bound of the uniform distribution, that is 1.

Another determinant of the bidding competition is the variance of the distribution of valuations. The larger this variance, the larger is on average the difference between the highest valuation and the second highest valuation and the larger is the economic rent to the winning bidder. But on the other side, increasing the variance of a distribution holding the mean constant, increases the second highest valuation as well. This has been shown for normal and uniform distributions (McAffee and McMillan 1987, p. 711). This implies that in

the case of the second-price sealed-bid and English auction the expected price will increase as well. Applying the revelation principle this holds for any optimal auction.

Collusion can occur by bidders forming a ring. A ring occurs when a group of bidders collude over their bids and submit only one bid through a member of the ring. In English auctions, a common method in practice is for the member with the highest valuation among the members of the ring to bid for the item without competition from his fellow cartel members. The member with the highest valuation for the object is found by conducting a fictitious English auction among members of the ring before the real auctioning of the object takes place. In the real auction, this bidder, if successful, will receive the bid to a price equal to the second highest valuation among those members of the auction not being part of the ring. The ring realizes a gain equal to the difference between the second highest valuation inside the ring and outside the ring. The successful member of the ring will pay a price equal to the second highest valuation among members of the ring, but has acquired the object for the ring at a price equal to the second highest valuation among non-members of the ring. This difference is distributed equally among members of the ring. Such behavior occurs in auctions for antiques, fish, timber, industrial machinery and wool (McAffee and McMillan, 1987, p.725).

Which auction design is most susceptible to such rings? A ring can only persist, if it does not pay for a member to cheat and/or cheating is easy to detect by members of the ring. In a second-price sealed-bid and English auction, a member cannot successfully cheat by getting a friend to bid on his behalf against the ring representative. This would only increase the price to be paid by the ring because the bidding member of the ring will in the English auction keep on bidding until his valuation, which is the highest among members of the ring, is reached respective in the first-price sealed-bid auction submit his valuation as a bid. Hence the ring is unlikely to be broken by cheating. In a first-price sealed-bid or Dutch auction there is much to gain by cheating on the ring. In this case the friend of a member of the ring just bids slightly higher than the representative bid of the ring and has a greater chance of getting the object cheaply than if the ring had not existed. Here the ring is more likely to be destroyed by cheats. Robinson (1985) showed that as long as cartel members share the same information, cartels are incentive compatible in the English auction, but not in the Dutch and the first-price sealed-bid auctions. This may explain the fact that the later auctions are often used in practice.

Another argument confirms the view that the English auction is particular sensitive to collusion among bidders. In an infinitely repeated game, a collusive outcome can be maintained as a non-cooperative equilibrium, if each of the players adopts a strategy of threatening to retaliate to any deviation from the collusive arrangement by reverting to non-cooperative behavior in future periods. This is the content of the folk theorem. For retaliatory strategies to be workable, it must be possible for the bidders to infer the other bidders' past action. If bids are made public, like in the English auction, this is the case. Accordingly it could pay for the seller not to give information to the bidders on the bids submitted.

What should a seller do if he believes he faces a cartel? According to Cassady (1967, pp. 228-30), reservation prices are commonly used to counter the activities of a ring. The anticartel reservation price increases with the number of cartel members, in contrast to the reservation pirce derived above. It has to be noted here, that the optimal size of the cartel is including all bidders. Furthermore, the anticartel reservation price is higher than the optimal reservation price in the absence of collusion.

Risk averse buyers

In the benchmark model we assumed risk neutral bidders. If bidders are risk averse, the optimal strategy in the second-price sealed-bid and English auction will not change. In the first-price sealed-bid and Dutch auction the bidder has to face the trade-off between increasing the probability to win and decreasing her rent due to a higher price to be paid, when increasing her bid. Risk-avers buyers will tend to increase the probability to win and such increase the price received by the seller. Accordingly the first-price sealed-bid and Dutch auction produces a larger expected revenue for the seller than the second-price sealed-bid and English auction.

With risk-avers buyers, the first price sealed bid and Dutch auction are not the optimal auctions, because they fail to maximize expected revenue for the seller when bidders are risk avers. The argument is the following: if the seller is risk neutral and the bidders are risk avers, there may be gains from trade in risk. The principles of the principal-agent models apply here.

The optimal auction with risk-avers bidders is very complicated (McAffee and McMillan 1987, pp. 719-720). The optimal auction in this case involves subsidizing high bidders who lose and penalizing low bidders. This is done by making the bidders' certainty equivalent payment positive for bidders with low valuation and negative for bidders with high valuation. This provides high bidders with more insurance than low bidders. Competition among high valuations will increase. Because the optimal auction with risk avers buyers is so complicated, requiring payments from some losing bidders and subsidizing others, it is unlikely to arise in practice. If the risk aversion is not very strong, the optimal auction is approximated by a sealed-bid auction with a bidding fee that is a decreasing function of the submitted bid.

In the case of independent and identical distributed valuations, the highest bid always comes from the person with the highest valuation of the good, because the bidders and accordingly the equilibrium strategies are symmetrical. We assumed risk-neutral bidders. In the case of bidders with different risk-aversion this will not hold. The allocation is no longer Pareto-efficient. Secondary markets will emerge and generate some of the rent missed by the non-efficient allocation. This view shows, that secondary markets may give the hint that the allocation due to the auction mechanism is not optimal.

Affiliated values and independent private values

In auction theory, all the assumptions of the benchmark model

- the bidders are risk neutral

- the independent private value assumption applies, that is bidders valuations are independent

- the bidders are symmetric, that is that the distribution underlying bidder's valuations is identical for all bidders

- payment is a function of bids alone, that means entrance fees etc. are excluded from consideration.

have been modified in several papers. McAffee and McMillan (1987) give an overview. Risk-avers bidders have been discussed already in this paper. Now, we will report shortly on the other results of auction theory. We will first discuss the assumption of asymmetric valuations.

So far we have assumed, that the valuations are drawn independently from an identical probability distribution function. If bidders draw their valuations independently from different distributions, revenue equivalence breaks down. The first-price sealed-bid and Dutch auction in general yield a different price from the second-price sealed-bid and English auction. Examples have been constructed which show, that the expected price may be higher or lower (McAffee and McMillan 1987, p. 715). We will not discuss this case more in detail, instead we will report on the results of auction theory in relaxing the assumption of independent private values.

In the case of offshore oil drilling licenses or spectrum licenses, the true value of the good is unknown to the bidders when submitting the bid, but there exists later some kind of an objective value, which is common to all buyers. Consider first the extreme case of the common value model, in which the bidders guess about the unique true value of the object to be sold. When the good being bid for has a common value, the phenomenon "winner's curse" can rise. The bidder with the highest estimate about the true value of the object wins and pays the highest price. Thus there is a sense in which winning conveys bad news to the winner, because it means that everyone else estimated the item's value to be less. But sophisticated bidders will take this into account in submitting the bid. Or to put it more technical, as in McAffee and McMillan (1987, p. 721): Suppose the ith bidder's information abut the items true value v can be represented by a number x_i, such that a bigger value of x_i implies a bidder true value v. Then

$$E(v| x_i) \geq E(v| x_i, x_i > x_j \text{ for all } j \neq i).$$

This result from probability theory shows, that the mere knowledge of the bidder, that she has won, will cause a naive bidder to revise downward her estimate of the item's true value. The rational bidder in a common value sealed-bid auction avoids becoming a victim of the winner's curse, She will set her bid according to what she estimated to be the second highest perceived valuation given that all other bidders are making this presumption.

The essential difference between the English auction on the one hand and the first-price and second-price sealed bid and to some extent the Dutch auction on the other hand is that the process of bidding in the English auction conveys information on the reservation prices of the other bidders. thus lessening the effect of the winner's curse.

A more general model is due to Milligram and Weber (1982), of which the common value model is a special case. This affiliated value model allows correlation among bidders valuations. When bidders' valuations are affiliated, the English auction yields a higher expected revenue than the second-price sealed-bid auction, which in turn leads to a higher expected revenue than the first-price sealed-bid auction, which leads to the same revenue as the Dutch-auction (Migrom and Weber 1982). The seller can increase his expected revenue by having a policy of making public any information he has about the item's true value. This will increase the value estimates of those bidders who perceive the item's true value to be relatively low, causing them to bid more aggressively.

In general, with the assumption that information will increase the bid because bidders become more informed on the true value of the object, it is in the interest of the seller to give all information available to the bidders. In the Nash-equilibrium expectations are confirmed and

accordingly it may be in the interest to report all information truthfully to the bidders. But in practice it may pay for the seller not to share all information with the bidders, in particular if information results in loser valuations of the good.

Multi-object auctions: simultaneous and sequential auctions

In the benchmark model and in most of the literature on auction theory it is assumed that only one object is sold. However in practice, several objects are sold either simultaneously or sequentially. We distinguish between:

- homogeneous units of a good, like treasury securities

- heterogeneous units of a good, like flowers or fish

- a unique good, like arts.

Only for unique goods, the several auctions taking place one after another for these goods may be regarded as a sequence of independent auctions. Though, even in this case the auctions in the sequence are linked together at least due to the budget constraints of the bidders. The link between subsequent auctions pursued in one auctioning event becomes closer with heterogeneous goods and closest with units of a homogeneous goods.

In the case of several homogeneous objects sold at an auction event, we distinguished between the uniform-price and the discriminatory auction. The uniform-price auction is equivalent to standard problem faced by monopolist with a given demand curve. The discriminatory auction has some resemblance to price discrimination, but the bid curve is not identical with the standard demand curve.

If several units of a homogeneous good are auctioned off, the sellers revenue and the number of items sold will depend on the order in which these goods are sold (Schotter 1974). In general, we expect prices to decline during the course of the auction event. The bidders with the highest valuations will get an item very early during the auction event and accordingly drop out. The longer the auction event goes on, the less bidders will remain active in bidding and the lower the price. Accordingly we expect a downward trend in auction prices. This is confirmed by empirical research (Becker 1985). Sophisticated bidders will take this into account. In an English auction with several units of the good sold subsequently, it is no longer a dominant strategy for each bidder to submit the true value as a bid. Depending on the expectations about the true valuations of the other bidders, the optimal bid of bidders with high valuations tends to be lower than the true valution.

From real to virtual auctions

Auction theory has analyzed the benchmark model and variations of it. The results show, that it depends on the characteristics of the bidders, which auction mechanism maximizes sellers revenue. Accordingly we would expect to find several auction mechanisms in practice.

Auction theory gives close attention to the characteristics of the bidders, but hardly any attention to the characteristics of the good being sold. Some goods are well suited to be auctioned off, while for others alternative distribution channels are better suited. This depends on the characteristics of the good to be sold. In the beginning we distinguished between:

- fungible and not fungible goods

- perishable and not perishable goods.

I am not aware on any theoretical work in auction theory taking care of the different character of the goods sold.

It is rather obvious, that in general non-fungible goods, which have to be inspected by the bidders to evaluate the value, are sold through the English or Dutch auction mechanism. On the other side, those goods, which are rather fungible, may be sold in sealed bid or in internet auctions.

In the case of perishable goods, it is important to have a very fast mechanism to allocate the good. Accordingly, perishable goods tend to be sold with the Dutch auction, while non-perishable goods tend to be sold with the English auction.

With the internet, the auction mechanism gained much more importance (Reichwald et. al. 2000). Sometimes internet auctions are just supplementing real auctions, like in the case of fish. Sometimes goods are sold on auctions where a posted price has been used in the past. Furthermore new auctions emerge, for example auctions between software agents.

In designing auctions in practice, the results of auction theory reported above are mostly neglected. A prominent example are the fish auctions. In real fish auctions, the Dutch auction is used, because it is the fastest auction mechanism. Fish is a highly perishable good and has to be sold as soon as possible. But using the Dutch auction system for fish auctions on the internet does not make sense. Auctions on the internet could be conducted simultaneously. Sticking to the old mechanism in a new environment results in inefficiencies.

The goal of this paper has been to introduce the models and the results of auction theory to a wider audience. The example of the fish auctions demonstrates well, that something can go wrong, if not enough attention is devoted to the optimal design of the auction mechanism. I hope, that the reader of this paper, in contrast to other people working on electronic commerce, has learned something from economic theory. A prominent example for the neglect of the results of auction theory gives the article by Klein (2000) in the "Handbook on Electronic Commerce" on auctions, an otherwise excellent article. I hope that the reader became aware, that there is a lot to gain from auction theory in designing auctions on the internet.

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